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Variable G and the Strong Equivalence Principle[1]

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A possible time variability of G, implying a violation of the strong equivalence principle, was first proposed by P. A. M. Dirac in 1937. Since such a feature cannot be accommodated within either Newton's or Einstein's theories, a new theoretical framework is needed. In this paper we review one such possible scheme, the scale covariant theory, within which the consequences of a variable G on geophysics, astrophysics, and cosmology can be treated consistently. The global verdict is that G *may* have varied by as much as a factor of 25 since the time of nucleosynthesis, without any disagreement emerging in any case. In spite of this result, we are not entitled to conclude from our analysis that a variable G has been shown to exist or that it is needed, but only that its variation is *compatible* with known data. The proof that G varies can in fact only come from direct observations. However, since the previous analyses had concluded that a $G(t)$ would entail severe discrepancies with known data, the reversal of the verdict is believed to be significant, since it may hopefully spur new observational interest in this basic problem.

1. CLOCKS, SCALE INVARIANCE, AND THE STRONG EQUIVALENCE PRINCIPLE

The advent of Einstein in theory brought what is generally believed to be the correct description of gravity. The success of the experiments testing Einstein's predictions leaves little doubt that Einstein gave us the correct physical interpretation of gravity as well as the theoretical tools to describe it from a classical point of view (Will, 1979). In the whole of physics, the success of general relativity (GR) vis à vis observations is comparable only to that of quantum electrodynamics (QED) in describing electromagnetic forces.

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Both GR and QED are "complete" theories, in the specific sense that they provide their own clocks: a *gravitational clock* and an *atomic clock*, each of them satisfying the dynamical equations of the respective theories.

A gravitational clock, marking a dynamical (or gravitational) time, is typically represented by two revolving planets whose period is governed by Kepler's third law $P \sim (GM/R^3)^{-1/2}$, a relation that follows from integrating the dynamic equations of motion. Analogously, an atomic clock (an hydrogen atom performing transitions) satisfies the dynamical laws ensuing from QED. It is important to stress once more that for a theory to be "complete" the clock should not be given from outside but provided by the theory itself.

The fact that we possess two theories that are very satisfactory within their domain of applicability does not tell us how they couple, and what to expect when we check the predictions of one dynamics (say gravitational phenomena), using a device governed by the other dynamics, for example an atomic clock. If we do not mix the two dynamics, the operational procedure is straightforward: we check the predictions of the Einstein (or Newtonian) mechanics using gravitational clocks, and those of QED using atomic clocks.

While we always use atomic devices to test QED, the same is not true when we check the predictions of GR. In fact, since 1955, when atomic clocks become available, we routinely use atomic, *not* gravitational clocks, to check the predictions of GR. Such mixing is of no consequence if the time intervals we deal with are very short. However, should the time-base become much longer, one may wonder whether such "mixing" may introduce other effects.

The concern is particularly relevant in cosmology, where one deals with far away objects which entail large fractions of the age of the universe. To be certain that our recording of gravitational phenomena with atomic clocks does not introduce "distortions," one should prove that atomic and gravitational clocks have been ticking at the same rate at all times during the expansion of the universe. This is, however, impossible if we measure $\Delta t_E/\Delta t_a$ (Δt_E and Δt_a represent the time intervals measured by dynamical and atomic clocks, respectively) *at one time only*, namely, today, an operation that gives us no guarantee that the value so obtained was the same in the past. It thus follows that our confidence in the cosmological conclusions based on the use of atomic clocks rests on the assumed outcome of experiments we have actually not performed.

Faced with this situation, we can think of two alternatives:

(1) We can postulate that "*an experiment (gravitational or otherwise) performed today yields results identical to those of experiments performed at any other time*." In other words, we may assume that atomic and gravita-

tional clocks have always been in the same ratio as today. This assumption (Will, 1979), known as the *strong equivalence principle*, SEP, is one of the pillars on which our knowledge about the past history of the universe rests.

(2) We can construct a *unified theory* of GR and QED. In this case, we would know the answer from first principles without the need of further assumptions.

Since we do not yet have such a theory, in spite of much effort, it may seem that possibility (1), the acceptance of the SEP, is the only alternative. Its simplicity, together with the fact that it has not yet led to any discrepancies with observations, have over the years created an aura of well-deserved "a posteriori credibility." It must in fact be admitted that, since the data we use in cosmology (in spite of being few in number) entail a large fraction of the age of the universe, there must be more than just "some truth" in the SEP. It is a remarkably good approximation if we further consider that we had no a priori guarantee that it would have worked when it was first adopted. It is indeed a good guess, once we further consider that we might have suspected just the opposite on the grounds that the Einstein equations are not invariant under scale transformations. By that we mean that if the length $ds=(g_{\mu\nu}(x)dx^{\mu}dx^{\nu})^{1/2}$ is a solution to the Einstein equations, the scaled length $ds'=\gamma(x)ds$ is no longer a solution of the same equations. Here, dx^{μ} is a dimensionless coordinate.

Einstein equations are therefore giving us an important hint by not being scale invariant: their solutions do not hold in just *any* system of units, but only in a specific one that we may naturally call the *dynamical (gravitational) system of units.*

Since the relation between gravity and electromagnetism is not known, there is no apparent reason why such a system of units should be identical with the one provided by atomic clocks. For this reason, one may have given little chance to the SEP of being correct, and yet it has turned out to be a good guess, at least so far. It logically follows that any possible violation of the SEP, if it exists, must be rather small, thus making its search and detection rather difficult.

Furthermore, one may ask the question: why do we want to investigate possible SEP violations? Is there a strong observational motivation? If not strong, the quality of the experimental evidence pointing to possible violations of the SEP is certainly improving, as the latest results on the moon's period suggest (see Section 2).

The case of the SEP is actually not unique in cosmology. An analogous situation occurs with the expansion of the universe, in which we believe for the same basic reasons we believe in the SEP, and which has also not been tested "directly." For this reason, Sandage (1974) proposed to measure the ratio θ_i/θ_m (θ_i being the isophotal and θ_m the metric angles), using the upcoming space telescope facilities. The function θ_i/θ_m vs. z is independent of q_0 and it behaves differently depending on whether we employ an expanding or a nonexpanding universe, for which the redshift is caused by mechanisms other than the expansion. Clearly, the results of such measurements will be of the utmost relevance to a very basic question in cosmology.

In the case of interest here, a check of the SEP would consist of measuring the (atomic) time evolution of the distance R and/or the period of P of a planet around the sun.

Since R and P are governed by gravity only, Einstein's equations predict R and P to be constant with respect to dynamical times. A nonnull observational result arrived at using atomic devices would imply that the two clocks are *not* identical and that the SEP is therefore violated. This type of experiment, conducted by I. I. Shapiro at MIT, and by R. Helling at JPL, is our best hope for an unequivocal answer (see Section 2).

While the observational search is being actively pursued, we believe that concomitant theoretical work should be carried out to further test and clarify the limits of validity of the SEP. What we have in mind is a middle-of-the road approach between the extremes (1) and (2) just discussed.

One can in fact conceive of a theoretical scheme that incorporates SEP-violating terms whose magnitude can then be estimated using presently available data. In other words, instead of assuming "a priori" that such terms are zero, we should inquire whether the data demand them to be so. It seems to us that the exercise is worthwhile even if the answer is that they must be zero. In fact, in that case we would have not only strengthened the reliability of the SEP, but also somewhat lessened the need for a more direct observational proof, which, as we have said, is far from simple to come by.

Such a phenomenological approach, begun in 1948 with a well-known paper by Teller (1948), has an interesting and instructive history.

The trick was to simulate the violation of the SEP by adopting a time-dependent gravitational constant G, as Dirac first suggested in 1937. This is an acceptable procedure. In fact, since G may be considered a "spring" of a gravitational clock (Kepler's third law), the possibility that an atomic clock may not keep pace with a dynamical one is equivalent to saying that G, *while constant with respect to dynamical units, may vary with respect to atomic clocks.* (It is a case of relative, not absolute, variation. We could equally well say that the "springs e and m" of an atomic clock may vary with respect to dynamical clocks. While correct, this point of view is of little operational use since we never use dynamical clocks.)

While the transition $G \rightarrow G(t)$ was correctly performed where G *explicitly* appeared, it was further assumed that the relations not containing G explicitly ought to be considered unaffected by the above transition and therefore safely employed. When that was done to analyze the past luminosity of the sun (Teller, 1948), the m vs. z relation for QSO (Barnothy and Tinsley, 1973) and the abundances of He and D (Barrow, 1978), discrepancies of alarming magnitude emerged, which, regrettably, may have had more than a marginal influence on those interested in checking observationally the SEP.

The discrepant results can, however, be shown to be the consequence of an incorrect assumption (Canuto and Hsieh, 1980a, b). Consider in fact Newton's law

$$F = G\frac{m_1 m_2}{r^2} = \nabla V(r) \tag{1}$$

and let us transform from a constant to a time-variable $G(t)$. The potential energy $V(r)$ becomes $V(r, t)$ and so energy is no longer conserved. It follows that relations like

$$\rho_\gamma \sim T^4, \qquad p \propto \rho^\gamma, \qquad M = \text{const} \tag{2}$$

that have a built-in energy conservation principle (in fact they come from the first law of thermodynamics) may no longer be used.

If G is not constant, it can be shown that the previous relations become

$$G\rho_\gamma \sim T^4, \qquad pG \sim (\rho G)^\gamma, \qquad MG = \text{const} \tag{3}$$

thus showing that the reason why many relations do not contain G explicitly is not because G has no right to be there, but rather because it has already been assumed to be constant. Therefore, *the G-independent forms, equation (2), should not be used in a G-varying context.* Regrettably, however, they were used in the previous tests. The reported discrepancies must therefore be interpreted as due to an inconsistent procedure rather than to the effect of $G(t)$, whose true implications remain at this point unknown.

The results of this discussion can be summarized as follows:

(A) Contrary to what was believed in the past there is no simple way to disprove possible violations of the SEP. It is a task much harder than originally suspected since it entails subtle (and difficult-to-track) changes in many conservation laws. (Canuto and Hsieh, 1980a, b)

(B) The substitution of G with $G(t)$, a legitimate if somewhat restricted "representation" of violations of the SEP, cannot be correctly carried out unless other relations are simultaneously modified. The use of the Newtonian scheme, with educated guesses at the possible changes caused by a $G(t)$, is in our opinion a risky way of proceeding.

(C) To be able to consistently appraise the effects of a $G(t)$, it is therefore advisable to first construct a theoretical framework that *allows* G to vary without incurring internal inconsistencies and only then carry out the desired tests.

Before we present such a formalism and the results of applying it to a host of different physical processes, let us present the best data relevant to the problem that are presently available.

2. THE DATA

To test the possibility that the strong equivalence principle (SEP) may be violated, we must show that

$$\Delta t_E / \Delta t_a = \beta_a \neq \text{const} \tag{4}$$

i.e., that atomic and gravitational clocks do not yield the same results while measuring the same phenomenon.

For that, consider two orbiting celestial objects whose period of revolution and separation are P and R, respectively. If the two objects can be considered point masses, Einstein equations (or their Newtonian limit) imply

$$P_E \sim (G_E M_E)^{-2}, \qquad R_E \sim (G_E M_E)^{-1} \tag{5}$$

Furthermore, it is known (Canuto and Hsieh, 1979a) that a direct consequence of the Bianchi identity is that

$$G_E M_E = \text{const} \tag{6}$$

so that from (5) we obtain

$$P_E \sim \text{const}, \qquad R_E \sim \text{const} \tag{7}$$

Since we want to keep the clocks separate, it is clear that (5)–(7), arising from the use of the Einstein equations only, must be understood to hold with respect to dynamical or gravitational clocks, thus justifying the subindex E. *With respect to these clocks, periods and distances are therefore predicted to remain constant in time.*

Suppose we now measure the period P (and the distance R) using *atomic clocks* instead, by sending radar waves to selected planets of the solar system, and then measuring their round-trip time. In so doing, we are recording the behavior of a quantity governed by gravity alone (over an extended period of time) using clocks whose dynamics is nongravitational. *If* our result turns out to be

$$P_a \sim \text{const}, \qquad R_a \sim \text{const} \tag{8}$$

(where the subscript a indicates the atomic nature of our measurements), then we would have found no changes in switching from atomic to dynamical times: *the SEP would therefore be confirmed.* This is indeed the kind of experiment I. I. Shapiro and R. D. Reasenberg of M.I.T. have been conducting in the last several years. The data available so far, as published by the authors (Reasenberg and Shapiro, 1978) are as follows:

Mercury:

$$\dot{P}_a/P_a = -(12 \pm 8) \times 10^{-11}/\text{yr}$$

Venus:

$$\dot{P}_a/P_a = -(12 \pm 12) \times 10^{-11}/\text{yr}$$

Mars:

$$\dot{P}_a/P_a = -(50 \pm 66) \times 10^{-11}/\text{yr} \tag{9}$$

Combined:

$$\dot{P}_a/P_a = -(30 \pm 18) \times 10^{-11}/\text{yr}$$

Average:

$$\dot{P}_a/P_a = -(12.4 \pm 6.6) \times 10^{-11}/\text{yr}$$

While the authors have remarked that systematic errors still exist which make them believe that the values are consistent with zero, they have also remarked on the fact that *each planet independently* yields a negative $\dot{P}_a$, which is precisely what other data on the Earth–Moon system also indicate. (It is important to remark that Shapiro, Helling, Adams, Canuto and Goldman are presently analyzing more data and that new results may be available soon.)

Let us now consider the data on the Moon. Contrary to the case of the planets just considered, the Earth and the Moon cannot be treated as a point masses, essentially because the Moon is very close to the Earth and the Earth is not an incompressible fluid. The net result is that the Moon generates tides which slow down the Earth's spin thus lengthening the day. The spin angular momentum lost by the Earth goes to the Moon and so does part of its rotational energy. The Moon acquires some energy and slowly frees herself from the Earth's bond. In this process, the Earth–Moon distance increases and the Moon's period becomes longer as time progresses.

Given now a fixed star in the sky, it is clear that the Moon will *not* occult it at the same time in each of its revolutions around the Earth, but progressively later each time. The recording of the occultation time can therefore serve as a device to reveal the lengthening of the period.

The most accurate result (Van Flandern, 1981), based on approximately 9000 such occultations, is ($P=2\pi/n$)

$$\dot{n}_a = -21.4 \pm 2.6 \tag{10}$$

where the units are arcsec cy^{-2}, 1 cy $=10^2$ yr. Methods based on lunar ranging experiments have also been extensively used by Dickey et al. (1980) and by Calame and Mulholland (1978). Their latest results are

$$\dot{n}_a = -23.6 \pm 1.5, \qquad -24.6 \pm 3.9 \tag{11}$$

The subscript a in (10) and (11) stands to remind us again that these measurements are based on the use of *atomic clocks*. It is important to notice that the above values fall within a small interval and a good agreement among them can therefore be claimed for the first time in many years.

Because of the fact that the Earth and the Moon cannot be treated as point masses, we cannot assume the validity of (7). In fact, it does not hold. Several values of $\dot{n}'_E$ are available in the literature. They are (Canuto, 1981)

$$\begin{aligned} \dot{n}'_E = -26.0 \pm 2.0, \qquad -28.5 \pm 3.1, \qquad -30.0 \pm 3.0 \\ -27.4 \pm 3.0, \qquad -30.6 \pm 3.1 \end{aligned} \tag{12}$$

where a prime indicates that gravitational clocks have been used.

The first three values are derived from analysis of ancient eclipses which were clearly recorded using the only clocks available then, namely dynamical ones, like the rotation or revolution of the Earth. While the errors are still large, there seems to be a tendency for $\dot{n}'_E$ to cluster around -28.5, whereas the $\dot{n}_a$'s cluster around -23.2. Since by (4) we also have

$$n_a = \beta_a n_E \tag{13}$$

it follows that

$$\frac{\dot{n}_a}{n_a} - \frac{\acute{n}_E}{n_E} = \frac{\dot{\beta}_a}{\beta_a} \tag{14}$$

where we have taken $\beta_a(t_0) = 1$. Since $n = 1.733 \times 10^9$ arcsec cy^{-1}, we deduce

$$\dot{\beta}_a/\beta_a = 3.05 \times 10^{-11}/\text{yr} \tag{15}$$

A more detailed analysis of the previous data must take into account the error of each value. When that is done and a least-squares fit analysis is performed, one obtains (Canuto, 1981) (in units of 10^{-11}/yr)

All values:

$$\dot{\beta}_a/\beta_a = 2.75 \pm 0.64$$

First $\acute{n}_E$ deleted:

$$\dot{\beta}_a/\beta_a = 3.41 \pm 0.54 \tag{16}$$

Second $\acute{n}_E$ deleted:

$$\dot{\beta}_a/\beta_a = 2.69 \pm 0.92$$

(The reason for the deletion of the first and second values is that they are not independent.)

At present, this is all the evidence we have for a nonnull $\dot{\beta}_a$. In spite of the errors bars still too large, it is, however, important to stress that the Moon occultation data are now yielding a value that not only is stable, in the sense that new data have been shown not to affect it, but is also very close to the two other values obtained with quite independent techniques. The conclusion seems to be that the atomic values for $\dot{n}_a$ are now converging and large changes are no longer expected to occur. (More data will reduce the error.)

The real uncertainties concern the dynamical values $\acute{n}_E$, since they still show more scatter than the atomic values. Our hope is to see a reanalysis of the ancient eclipse data (Stephensen, 1981).

While the Earth–Moon system will certainly be further analyzed and the values of $\dot{n}_a$ and $\acute{n}_E$ further refined, it is clear that there will always be doubts as to how reliable is the extraction of small effects of cosmological nature from such dynamically complicated systems. It is for this reason that it is generally believed that the work of Shapiro and Helling et al. holds the real key to the problem (see Canuto and Goldman, 1982).

3. ATOMIC AND GRAVITATIONAL CLOCKS

The intriguing data just presented are not the only clue that the SEP requirement

$$\beta_a = \text{const} \tag{17}$$

may be violated on time scales of the order of 10^{-11}/yr. As noted in Section 1, we also have some general theoretical considerations which we shall now state in a more precise form and which will help us to define the distinction between atomic and gravitational clocks.

The SEP requirement (17) is tantamount to assuming that we do not have two distinct clocks but only one, $\Delta t_E = \Delta t_a$. As we have already stated, clocks are the macroscopic manifestation of a deeper feature, the interactions. If (17) were an exact law, it ought to be imprinted in the nature of such interactions and consequently deducible from them. And this is precisely our point: the nature of gravitational interactions, as described by the Einstein equations, does not bear out this expectation. This can be seen as follows. Consider Einstein equations written, as they should be, in dynamical units (denoted by the index E)

$$R^E_{\mu\nu} - \tfrac{1}{2} g^E_{\mu\nu} R^E = -8\pi G_E T^E_{\mu\nu} \tag{18}$$

The solution to these equations will be denoted by ds_E,

$$ds_E = c\, dt_E = g^E_{\mu\nu}\, dx^\mu\, dx^\nu \tag{19}$$

For the dynamical clock, marking Δt_E, to be identical to the atomic one, marking Δt_a, as (17) demands, it would have to be true that Einstein equations retain the form (18) even with respect to ds_a

$$ds_a = c\, dt_a = g^a_{\mu\nu}\, dx^\mu\, dx^\nu \tag{20a}$$

where

$$g^E_{\mu\nu} = \beta_a^2 g^a_{\mu\nu} \tag{20b}$$

This is, however, not so, as is easy to prove. To do so, let us perform in (18) the transformation from dynamical to atomic units $ds_E = \beta_a ds_a$. We obtain (Canuto et al., 1977)

$$R^a_{\mu\nu} - \tfrac{1}{2} g^a_{\mu\nu} R^a + f_{\mu\nu}(\beta_a) = -8\pi G_a T^a_{\mu\nu} \tag{21}$$

where $(\beta_\mu \equiv \beta_{,\mu})$

$$\beta^2 f_{\mu\nu}(\beta) = 2\beta\beta_{\mu;\nu} - 4\beta_\mu\beta_\nu - g_{\mu\nu}(2\beta\beta^\lambda_{;\lambda} - \beta^\alpha\beta_\alpha) \tag{22}$$

An extra term $f_{\mu\nu}(\beta)$, originating from the left-hand side of the Einstein equations, has appeared which makes (21) differ from (18). To make them equal, we need to use (17), in which case $f_{\mu\nu} = 0$.

It is therefore clear that the SEP "forces," so to speak, the identity between Δt_E and Δt_a, by making sure that the Einstein equations *do look* the same in both units. This is, however, not a property intrinsic to the Einstein equations, but one imposed from outside. We shall therefore define the following:

Gravitational Units (called E units, E for Einstein) as the units marked by a dynamical clock (a planet and a star) and in which the Einstein equations retain exactly their original form as consequently do all the gravitational expressions ensuing therefrom. For this reason, we can claim that we do not change the Einstein theory, as for example Brans and Dicke did. Indeed, we do not: we only specify that such equations must be understood to hold in one particular system of units, the one marked by a dynamical clock.

Furthermore, in E units,

1. Planetary periods are constant, equation (7);
2. Macroscopic masses (total and rest masses) are constant, $M_E = \text{const}$; however, *microscopic masses need not be*;
3. Macroscopic bodies move along geodesics $u^\mu_{;\nu} u^\nu = 0$;
4. The gravitational constant G_E is constant.

Let us now define *atomic units* (denoted by a subscript a) as those units in which

1. All microscopic quantities like

$$m_e, \qquad \hbar_a, \qquad e_a \tag{23}$$

are constant;
2. Schrödinger (and Dirac) equation holds true;

3. The period of an electromagnetic clock is constant; *however*,
4. A planetary period is not constant with respect to A units;
5. An electromagnetic clock has a nonconstant period with respect to E units.

We are therefore searching for a theory that, while it yields

$$p_a, P_E \sim \text{const} \tag{24}$$

also yields

$$p_E, P_a \neq \text{const} \tag{25}$$

where p and P are the periods of an electromagnetic and gravitational clock, respectively. The subscripts a and E indicate the type of clock used.

Clearly, a result like $p_E \neq$ const is of little operational interest since we clearly do not use dynamical clocks to measure atomic transitions, much as we no longer use them to measure gravitational phenomena. However, in spite of that, the requirements $P_E =$ const and $p_E \neq$ const are indispensable requirements for the internal consistency of the theory.

Equations (24) and (25) represent a *broken symmetry*, i.e., atomic and gravitational clocks are not the same, but

$$p_a/p_E \sim \beta_a, \qquad P_a/P_E \sim \beta_a^{-1} \tag{26}$$

Quantities in the two systems of units transform with respect to one another following (4) or for any quantity Λ

$$\Lambda_E = \beta_a^{\pi(\Lambda)} \Lambda_a \tag{27}$$

For future reference, we shall quote some important relations. If $\Lambda \equiv M = mN$, and since $\pi(N) = 0$, we derive

$$m_E \sim \beta_a^{\pi(m)}, \qquad M \sim \beta_a^{-\pi(m)}, \qquad N \sim \beta_a^{-\pi(m)} \tag{28}$$

since by definition

$$m_a, \quad M_E \sim \text{const} \tag{29}$$

While in E units, macroscopic masses are constant by definition, microscopic masses are not so, unless $\pi(m) = 0$. In A units, the role of the masses is inverted.

Before we proceed, it is important to give some physical interpretation to the previous choices as well as to contrast their implications with those of other approaches. With two systems of units at our disposal, one could have chosen the atomic units as the ones in which *all* atomic phenomena are described by the standard relations, i.e., β_a should not enter in *any* atomic expression. However, we have not done so. Instead, we have chosen the gravitational units as our "*fiducial units*," by adopting the point of view that gravity is fully described by Einstein equations, provided one reads them in E units. This choice is to be contrasted with the one made by Brans and Dicke, in whose theory *all* atomic expressions, *at the level of one particle as well as that of many particles*, were assumed to be described by the standard expressions. Gravitational expressions were, however, changed, being affected by the Brans–Dicke (BD) scalar function ϕ.

In our theory, what we assume is that *microphysics* should remain unchanged only at the *one-particle level*, i.e., the Schrödinger and Dirac equations should not be affected by β_a. However, many-body expressions, such as for example an equation of state, the Stefan–Boltzman law for a photon gas, etc. may be affected by β_a. *Many*-body systems are therefore not the simple *sum of one-body properties*. The transition from one to many particles is a delicate process since in the present framework we must allow for a nonconstant baryonic number N, a possibility a priori excluded in the BD approach.

4. GENERAL UNITS—BROKEN SYMMETRY

The previous arguments indicate that we must construct a theory with a broken symmetry between atomic units (AU) and Einstein units (EU), i.e., a theory that satisfies the requirements (1) through (4) and (1) through (5) of Section 3.

In order to do so, we shall make use of the gravitational Lagrangian presented in Canuto et al. (1977), to which we shall add the matter Lagrangian (Canuto and Goldman, 1982a)

$$\mathfrak{L}_m = \int \mu \beta^{2-g}\, ds \tag{30}$$

where μ is the general symbol for a mass and where $G \sim \beta^{-g}$. It is easy to see that the power of $\mathfrak{L}_m$ is zero.

A word of explanation is necessary. In (30), as well as in previous work, we have used the function β to write the equation in *general units*, i.e., β is unobservable. It is only when we go to AU and EU that β acquires a physical meaning. In fact we have

AU:

$$\beta = \beta_a$$

EU:

$$\beta = 1 \tag{31}$$

We shall therefore first write our equations in general units and then specify them to either AU or EU. For example, from (30) we deduce that the equation of motion is (Canuto and Goldman, 1982) *in general units*,

$$u^{\mu}_{;\nu}u^{\nu} + \frac{(\mu\beta^{2-g})_{,\nu}}{(\mu\beta^{2-g})}\Delta^{\mu\nu} = 0 \tag{32}$$

where

$$\Delta_{\mu\nu} = u_{\mu}u_{\nu} - g_{\mu\nu} \tag{33}$$

Given this addition to $\mathfrak{L}_g$ presented in Canuto et al. (1977), let us now show how we can construct the two clocks.

5. GRAVITATIONAL MACROSCOPIC CLOCK[3]

In this section we shall show how using (30) we can satisfy some of the conditions of Section 3. Consider first a macroscopic object of mass M, i.e., $\mu = M$. Since in general units and because of (29)

$$M_E = M\beta^{1-g} = \text{const} \tag{34}$$

equation (32) reduces to

$$u^{\mu}_{;\nu}u^{\nu} + (\beta_{,\nu}/\beta)\Delta^{\mu\nu} = 0 \tag{35}$$

which further specifies to

EU: $\beta = 1$

$$u^{\mu}_{;\nu}u^{\nu} = 0 \tag{36}$$

AU: $\beta = \beta_a$

$$u^{\mu}_{;\nu}u^{\nu} + (\beta_{a,\nu}/\beta_a)\Delta^{\mu\nu} = 0 \tag{37}$$

[3]See Canuto and Goldman (1982a).

with the solutions for the period P given by

$$P_E = \text{const} \tag{38}$$

$$P_a = \beta_a^{-1} P_E \sim \beta_a^{-1} \tag{39}$$

which indeed proves two of the requirements (24) and (25). It is important to add that in order to solve (35), we have used the Scharszchild metric solution of the Einstein equation in EU, namely,

$$R_{\mu\nu}^E - \tfrac{1}{2} g_{\mu\nu}^E R = -8\pi G_E T_{\mu\nu}^E \tag{40}$$

6. ELECTROMAGNETIC CLOCK[4]

Let us go to the second part of our problem, i.e., to show how the other two conditions in (24) and (25) can also be satisfied. To that end, let us consider a classical model for an electromagnetic clock consisting of an electron and a proton. The most general "form-invariant" action is given by

$$I = \sum_{e,p} \int \mu\beta^{2-g}\, ds + (16\pi)^{-1} \int \beta^{2-g} F_{\mu\nu} F^{\mu\nu}\, ds$$

$$+ \sum_{e,p} \int \beta^{2-g} e A_\mu u^\mu\, ds \tag{41}$$

where the electromagnetic tensor $F_{\mu\nu}$ is related to A_μ by

$$\beta^{1-g/2} F_{\mu\nu} = \left(\beta^{1-g/2} A_\mu\right)_{,\nu} - \left(\beta^{1-g/2} A_\nu\right)_{,\mu}$$

Since $2\pi(F_{\mu\nu}) = 2\pi(e) = 2 - \pi(G) \equiv 2 - g$, it is easy to see that $\pi(I) = 0$. Clearly $\pi(F_{\mu\nu}) = \pi(A_\mu)$. From (41) we now obtain

$$\left(\sqrt{g}\, F^{\mu\nu} \beta^{1-g/2}\right)_{,\nu} = 4\pi \int e\beta^{1-g/2} \delta^4(x^\alpha - z^\alpha)\, dz^\mu \tag{42}$$

$$u^\mu_{;\nu} u^\nu + \frac{\left(\mu\beta^{2-g}\right)_{,\nu}}{\left(\mu\beta^{2-g}\right)} \Delta^{\mu\nu} = \left(\mu\beta^{2-g}\right)^{-1} \Phi^\mu_\nu u^\nu \tag{43}$$

[4] See Canuto and Goldman (1982a).

where

$$\Phi_{\mu\nu}=\left(eA_{\mu}\beta^{2-g}\right)_{,\nu}-\left(eA_{\nu}\beta^{2-g}\right)_{,\mu} \tag{44}$$

The requirement that in AU

$$\beta=\beta_a, \qquad \mu=m_a=\text{const}, \qquad e=e_a=\text{const}$$

the period of the electromagnetic clock be independent of β_a, i.e., constant, can be achieved *only if we choose*

$$g=2, \qquad G=\beta^{-g}G_E \tag{45}$$

in which case (42) and (43) reduce to the standard expressions, with the corresponding well-known solution

$$p_a=2\pi\frac{m_a^2 l_a^3}{e_a^4}=\text{const} \tag{46}$$

The requirement $p_a=$ const has forced us to choose a gauge, i.e., a relation between β and G. This is a most welcome feature since the theory as formulated and used so far had to use the value of g, as a free parameter. This is no longer the case. We still have to show that $p_E \neq$ const. To that end, let us consider (42) and (43) in EU, i.e., with

$$\beta=1, \qquad \mu=m_E=m_a\beta_a^{-1}, \qquad e=e_E=\text{const} \tag{47}$$

where we have used (28) and (45), as well as the fact that with $g=2$, the charge is constant in any units. Equation (42) therefore retains the standard form we are used to, whereas (43) does not. It becomes

$$u^{\mu}_{;\nu}u^{\nu}+\frac{\left(\beta_a^{-1}\right)_{,\nu}}{\left(\beta_a^{-1}\right)}\Delta^{\mu\nu}=\frac{e_a}{m_a}\beta_a F^{\mu}_{\nu}u^{\nu} \tag{48}$$

whose left-hand side is formally identical to that of (37). It is easy to show that

$$p_E=\beta_a p_a \sim \beta_a \tag{49}$$

i.e., electromagnetic clocks do not have constant period in EU.

With this last result, we *have therefore shown that the fundamental requirements of the theory can indeed be met.*

At this point, we may notice that (38) and (39) were already contained in Canuto et al. (1977). However, they represent only a necessary condition for the theory to be complete. *Equations* (45) *and* (49) *provide the remaining conditions for a two-times theory to be meaningful.*

In fact, while the operationally interesting relations are (38) and (39), since they are the ones used in the analysis of planetary periods, it is indispensable to show that the clock used to measure P_a, namely, the atomic clock, has a period that is constant in time.

7. THE WEAK EQUIVALENCE PRINCIPLE[5]

In achieving the above results, a fundamental role was played by the form of the action (30) and by the equations of motion ensuing from it. It has in fact been possible to choose the parameter g so as to have the following in AU:

Macroscopic bodies:

$$u^{\mu}_{;\nu}u^{\nu}+(\beta_{a,\nu}/\beta_a)\Delta^{\mu\nu}=0 \tag{50}$$

Microscopic bodies:

$$u^{\mu}_{;\nu}u^{\nu}=0 \tag{51}$$

telling us that one particle and many particles do not follow in time the same trajectory, the difference being due to the β_a term which reveals its presence over long periods of time. This conclusion may seem to constitute a violation of the weak equivalence principle (WEP).

It is our contention however that *none* of the present experimental setups devised to test the WEP as usually formulated has been directly used to test (50) and (51). In fact, the Eotvös–Braginski–Dicke (Will, 1979) experiments have shown that two bars of Al and Au follow the same trajectory, thus proving that the "stuff" they are made of has no influence on their motion. This well-established fact is, however, not directly usable to test the simplest of the consequences of (50) and (51), namely, that *over a long period of time*, one proton and N protons follow different paths. We therefore believe that equations (50) and (51) represent a very "sui generis" form of violation of the WEP, *if any at all*, predicting in fact a situation not covered by the experiments performed so far. We may add that macroscopic and microscopic objects separately do satisfy the WEP since the extra terms

[5]See Canuto and Goldman (1982a).

β_a do not depend on the mass of the macroscopic objects and therefore cancel out when two objects like bars of Au and Al are compared.

We would like to go further and present another point of view. While observationally testable consequences of (50) and (51) must certainly be searched for, it is our contention that *if* the data on the moon and on the planets *do reveal* that $\dot{n}_a$ and $\dot{n}_E$ are indeed different, that very fact is already a proof that (50) and (51) are correct. Alternatively, *we can say that the different behavior in time predicted by* (50) *and* (51) *will reveal itself as a difference in planetary periods.*

In fact, we may think of an atomic clock and of a gravitational clock (the Earth–Moon system), as two "objects" moving in space-time, following two given trajectories. If the two systems do not follow the same path as time evolves, it seems clear that charting the time evolution of the macroscopic object with the meter stick provided by the microscopic object cannot yield constant results, as the two span different lengths.

We therefore believe that far from representing an unexpected result, equations (50) and (51) are but an alternative way of interpreting the lunar and planetary data, a point of view which stresses the *difference in the trajectories* rather than the *difference in the behavior of the two clocks*, as traditionally done.

These two ways of interpreting the data are, in our opinion, equivalent, although the different behavior of the clocks has so far been the one almost exclusively referred to in this context.

In conclusion, if the data on the moon are confirmed, we believe we will have a proof that one and many particles follow in time different trajectories, a fact that does not contradict in any way the WEP as usually formulated.

In this connection, it is important to remark that once (50) and (51) are solved for the quantity of interest (a period, for example), the ratio of the two solutions is a function of β_a, a quantity which, being normalizable to unity at any one given time, cannot be revealed with *one* experiment at *one* time. Only two or more experiments over as large as possible a time span can reveal the presence (or absence) of β_a and therefore test (50) and (51).

8. THE BARYONIC NUMBER

Since

$$\pi(G) = g = 2 \tag{52}$$

it follows that

$$\pi(m) = -1 \tag{53}$$

since the quantity GM/c^2 has the dimensions of a length and therefore transforms like $\Delta t_E = \beta_a \Delta t_a$. Using (28), we further conclude that

$$N \sim \beta_a \tag{54}$$

i.e., that the baryonic number is not conserved. It must be stressed at this stage that (54) is a consequence of the theory and not an a priori requirement. We have set up a theoretical framework to accommodate the basic requirement that atomic and gravitational clocks should run at different rates. It turns out that this may be achieved only if G and N are no longer constant. Equation (54), perhaps the one least expected, can neither be proved nor disproved at present.

It may perhaps be of interest to stress that (54) marks the most important difference between this framework and the Brans–Dicke theory, which assumes N constant.

9. AN EQUATION FOR β_a[6]

In this theory β_a has been treated as an external input that enters into the various physical relations but whose dynamics is not determined within the present framework. This does not, however, prevent one from exploring in a consistent manner the implications of a nonconstant β_a on various physical and astrophysical phenomena.

The situation parallels that encountered in classical fluid dynamics, which contains parameters such as viscosity and diffusion coefficients which are also not determined by the theory but rather treated as external inputs. This does not prevent one from carrying out hydrodynamical calculations and comparing the results with experiments.

A possible way toward a completion of the theory would be to regard β_a as a *local space-time field* and try to include an action for it in the general action. This would lead to a coupling of β_a to local matter sources. If so, gravity, even in EU, would no longer be described by the standard Einstein equations, since source terms arising from the β_a action would now be present. The net result would be a Brans–Dicke-type theory, with the consequence that solar-system experiments would constrain the time variability of β_a to some orders of magnitude smaller than the one suggested by the lunar data. The order of magnitude of $\dot{\beta}_a$ as from the lunar data strongly suggests that the violation of the SEP, manifested via a nonconstant β_a, is related to the *global properties of the universe*. It therefore seems inescapable

[6]See Canuto and Goldman (1982a).

that a local Lagrangian approach is inappropriate, even in principle, for determining β_a.

A more fundamental theory (at the quantum level presumably) is required in order to construct a dynamics for β_a and at the same time explain the mechanism of baryon number nonconservation and so the nature of the transition from microscopic to macroscopic physics. Such an approach is still missing.

In the sections that follow we shall present a summary of the tests performed so far in which β_a has been treated as an external input.

10. THE TEST OF THE THEORY PERFORMED SO FAR[7]

Before we go into a detailed description of the results of our analysis, we shall change the notation slightly. Since we shall be dealing almost exclusively with *atomic units*, we shall drop the subindex a in order to simplify the notation.

A. Gravitational Tests (Canuto et al., 1977). Since in E units, the Einstein's equations remain unchanged, we can borrow the results from GR and scale them to atomic units. We obtain

(a) Deflection of Light. We obtain in atomic units

$$\Delta\phi = 4(GM/R) \tag{55}$$

where M is the mass of the sun and R its radius. Because of the relation $\beta GM = \text{const}$, we have

$$\Delta\phi = \Delta\phi_0 \frac{1}{\beta}\left(\frac{R_0}{R}\right) \tag{56}$$

where $(\Delta\phi)_0$ is value measured today. The radius of the sun depends on both atomic and gravitational forces. Contrary to purely gravitational distances, which are constant in Einstein units, the radius $R_E = \beta R$ need not be constant. Therefore

$$\Delta\phi \neq \Delta\phi_0 \tag{57}$$

thus making the deflection of light experiment suitable in principle to reveal the presence of β_a.

[7]See Canuto et al. (1977, 1979b); Canuto and Hsieh (1979); and Canuto and Owen (1979).

(b) The Radar Echo Delay. The atomic (*maximum*) proper-time delay can be derived to be

$$\beta\Delta t = \Delta t_0 - 8(GM)_0 \ln(\beta R/R_0) \tag{58}$$

where again R is the sun's radius at a time t and the index 0 indicates that the quantity is measured today.

(c) Planetary Orbits (Canuto, Hsieh, and Owen 1979a; Canuto et al., 1977). Since in Einstein units, planetary distances R_E and periods P_E are predicted to be constant, equation (8), it follows that if measured with atomic clocks, the result is

$$\frac{\dot{R}}{R} = -\frac{\dot{\beta}}{\beta}, \qquad \frac{\dot{P}}{P} = -\frac{\dot{\beta}}{\beta} \tag{59}$$

Using again the lunar data for $\dot{\beta}$, we predict a decrease in time of both the distance and the period of all planets, barring of course tidal effects. We may add that "the shrinkage of the size of the solar system" indicated by (59) may be a misleading way of presenting the results. What actually happens is that our measuring devices, the "atomic clocks," speed up with respect to gravitational clocks, so that periods seem shorter. The experiments by Shapiro and Helling et al. mentioned before measure directly P and R.

B. Thermodynamics *(1) Particles.* (Canuto and Hsieh, 1979). The fundamental relation on which to construct thermodynamical relations is equation (4.7) of Canuto and Hsieh (1979), i.e.,

$$dQ = p\,dV + dU + [(1-g)\rho + 3p]V d\phi \tag{60}$$

with $\phi = \ln\beta, U = \rho V$. Equation (60) is the "generalized first law of thermodynamics" to include β and G. If we separate ρ into $\rho_0 + u$, where ρ_0 is the rest mass energy density, then equation (60), for $dQ = 0$, can be integrated to yield

$$\beta G \rho_0 V = \beta GM = \text{const} \tag{61}$$

Together with

$$\beta^2 G = \text{const} \tag{62}$$

Eq. (61) can easily be derived from scaling. For the u part of ρ, we get an equation similar to (60), which can be easily integrated to yield (Canuto and Hsieh, 1979)

$$N_p \sim \beta, \qquad TV^{\Gamma}\beta^{3\Gamma} = \text{const} \tag{63}$$

where we have used the "definitions" $pV = NkT$, $\Gamma U = p$. Eliminating T in the expression for p leads to ($\gamma = 1 + \Gamma$)

$$p \propto \left(\frac{N}{V}\right)^{\gamma}\left(\frac{G}{\beta^2}\right)^{\gamma-1} \tag{64}$$

This relation clearly indicates that the polytropic expression $p \propto \rho_0^\gamma$ is no longer valid when G varies. As discussed earlier (Canuto, 1981), the adoption of $p \propto \rho_0^\gamma$ in a variable G scheme was justified on the grounds that it is an atomic relation, therefore immune from G. We have already stressed that while a single atom is expected to be independent of β (or G), an equation of state, being a many-body system, must depend on G because of the relation (61).

(2) Radiation (Canuto and Goldman, 1982b). For radiation, it was shown that equation (60) holds, with $\gamma = 4/3$, i.e., $3p_\gamma = \rho_\gamma$. We obtain

$$N_\gamma \sim \beta^0, \qquad \rho_\gamma \sim T^4, \qquad TV^{1/3} = \text{const} \tag{65}$$

It is important to note that the kinetic temperature T, defined to satisfy the relation $p = (N/V)kT$, is such that the average kinetic energy per particle $\langle \varepsilon \rangle \sim kT$. The same relation holds for the average energy of a photon $\langle \varepsilon \rangle \sim kT$.

C. Astrophysics (Canuto and Goldman, 1982b). *(1) Luminosity and Effective Temperature of the Sun*. Most astrophysical applications rest on the use of the hydrostatic equilibrium equation, whose expression

$$-\frac{dp}{dr} = G\frac{m(r)\rho(r)}{r^2} \tag{66}$$

can be shown to be valid in both atomic and gravitational units (Canuto et al., 1977). Another relation, also playing a fundamental role, is the radiative transfer equation which was shown to take the form

$$-k\rho L(r) = 4\pi r^2 \frac{d\rho_\gamma}{dr}\left[1 - \frac{1}{k\rho}(2+g)\frac{\dot\beta}{\beta}\right]^{-1} \tag{67}$$

where k is the stellar opacity. Neglecting the term $\dot{\beta}/\beta$, small compared with the transit time of radiation, using (65) for ρ_γ, and employing the constraint (61), we obtain the exact result

$$L \sim \frac{G^{5/2}}{k} \tag{68}$$

different from Teller's result

$$L \sim \frac{G^4}{k} \tag{69}$$

Using again (65), we find for the sun's effective temperature T_* the expression

$$T_* \sim \left(\frac{G^{5/2}}{k} \frac{1}{R^2} \right)^{1/4} \tag{70}$$

where R is the sun's radius.

(2) Effective Temperature of the Earth (Canuto and Goldman, 1982b). The knowledge of L can be used to calculate the influence of G on the Earth's effective temperature, $T_{\rm eff}$. The result is

$$T_{\rm eff} \sim \frac{G^{3/8}}{k^{1/4}} \tag{71}$$

where the Earth–Sun distance scales like β^{-1}, since it is determined solely by gravity.

(3) The Chandrasekhar Mass. Using the equation of state $p \sim \rho_0^\gamma$, it is well known that one obtains, for $\gamma = 4/3$, the so-called Chandrasekhar mass M_c

$$M_c = (\hbar c/G)^{3/2} m^{-2} \tag{72}$$

which may lead one to believe that M_c scales in general like $G^{-3/2}$. This is, however, incorrect in the present theory since masses should scale like $M \sim (\beta G)^{-1}$, as the general relation (61) indicates. The reason for the discrepancy is that $p \sim \rho_0^\gamma$ is no longer valid in a G-varying framework. Using (64) instead, it is found that

$$\beta G M_c \sim h^{3/2} m^{-2} \tag{73}$$

Since in atomic units the right-hand side is constant by definition, the expected result follows.

(4) Luminosity of White Dwarfs. Much concern has been voiced in the past about the possibility that a variable G may lead to a luminosity of white dwarfs in disagreement with observations. The somewhat strange results published on this subject (negative luminosities for example) were due to the use of the incorrect p vs. ρ relation. If (64) is employed, together with the definition of luminosity $-L = Ed\ln(\beta GE)/dt$, where $E = GM^2/R$, the following result holds:

$$L \sim (1/\beta G)L_{st} \tag{74}$$

Since we can only determine L at one time, namely, today when $\beta G = 1$, the luminosity L is clearly identical to the standard one.

(5) The Age of Globular Clusters (Canuto and Goldman, 1982b). One interesting possibility of detecting effects due to G and β is through the age of globular clusters, which may be substantially reduced. Using (68), we find that the lifetime $\Delta t_{st}(G = \text{const})$ is related to $\Delta t(G \neq \text{const})$ by the relation

$$\Delta t_{st} = \int_{t_0 - \Delta t}^{t_0} dt\, G^{5/2}(t)\left(\frac{k_{st}}{k}\right) \tag{75}$$

It is clear at this point that any further evaluation is not possible unless we know the β dependence of k and R, i.e., the functions

$$R = R_{st}r(\beta), \qquad k = k_{st}f(\beta) \tag{76}$$

are given: For details, see Canuto and Goldman, 1982b.

D. The Radius of the Earth (Canuto, 1981). A great deal of interest in the possible variation of G was due to its possible effects on the radius of the Earth. The literature on the subject is long and controversial, and for a full description we refer the reader to Canuto (1981). The generally accepted conclusion has been that if G was larger in the past, the radius of the Earth had to be necessarily smaller, therefore we had an expanding Earth. In fact, substituting an equation of state of the form $p \sim \rho_0^\gamma$ into (66) yields, together with the assumption of a constant mass,

$$R \sim G^{-1/3\gamma-4} \tag{77}$$

which is at the basis of all qualitative statements that a larger G implies a smaller Earth. The situation clearly changes in the present framework, since the equation of state is no longer $p \propto \rho_0^\gamma$. Insertion of (64) yields

$$R \sim GM \sim 1/\beta \sim G^{1/2} \tag{78}$$

i.e., a larger G implies a larger R. Equation (78) is, however, not completely acceptable since it implies that $R_E = \beta R$ is constant. Now R_E is not determined solely by gravity (like a planetary distance) and therefore it cannot be expected to be constant in Einstein units. The problem is extremely complex and only an approximate solution is possible (Canuto, 1981). In order to avoid the result $R \sim 1/\beta$, we have to introduce atomic interactions. For that, we introduce the equation of state

$$p = \frac{N}{V}kT\left[1 + B(T)\frac{N}{V} + C(T)\frac{N^2}{V^2} + \cdots\right] \tag{79}$$

Using $B(T) \sim T^{b-1}$ and (62), we finally obtain

$$p \propto \rho^\gamma \left(\frac{G}{\beta^2}\right)^{\gamma-1} - A\rho^{\gamma_*}\left(\frac{G}{\beta^2}\right)^{\gamma_*-2} \tag{80}$$

where A is a numerical constant. Inserting (80) into (66), we finally obtain for radius R the following expression:

$$R \sim (GM)^r \tag{81}$$

with

$$r - 1 = \alpha(2+g)\left[3\gamma - 4 - 3\alpha(\gamma_* - \gamma)\right]^{-1} \tag{82}$$

An equation of state of the form (80) was proposed by Birch, with $\gamma = 7/3$ and $\gamma_* = 5/3$. Taking $\alpha = 1$ and $g = 1$ (i.e., $M = \text{const}$), $r = 1.6 > 0$, so that

$$\dot{R}/R = -r\dot{\beta}/\beta < 0 \tag{83}$$

The present theory therefore predicts a decrease in time of the Earth's radius as due to G. It is clear that other effects of geophysical nature may overcompensate the effect of G and alter the result entirely. However, using for $\dot{\beta}/\beta$ the value determined before, we conclude that 400×10^6 yr ago, equation (83) predicts

$$R = (1.02 \pm .004)R_0 \tag{84}$$

in good agreement with recent paleomagnetic data

$$R = (1.02 \pm 0.028) R_0 \tag{85}$$

thus suggesting that perhaps the effect of a variable G is indeed of major importance and that the Earth is after all shrinking.

E. Cosmology (Canuto and Goldman, 1982b; Canuto et al., 1979b; Canuto and Owen, 1979). *(1) The 3°K Black-Body Radiation*. The observed 3°K background radiation has long been considered to be a remnant of equilibrium radiation from an earlier epoch. We briefly review the well-known chain of reasoning leading to the above conclusion, so as to facilitate subsequent discussion in the scale covariant framework:

(a) As the universe expands, individual photons suffer a frequency shift given by

$$\nu_0 / \nu = R_0 / R \tag{86}$$

(b) Standard conservation law gives for adiabatically expanding radiation

$$\rho_\gamma R^4 = \text{const} \tag{87}$$

If the radiation consists of noninteracting photons, the above conservation law holds for each spectral interval:

$$d\nu \, \rho_{\gamma\nu} R^4 = \text{const} \tag{88}$$

(c) For radiation in equilibrium, the spectral distribution is given by

$$\rho_{\gamma\nu} \sim \nu^3 [\exp(\nu/kT) - 1]^{-1} \equiv \nu^3 f(\nu/T) \tag{89}$$

If the radiation subsequently becomes a gas of noninteracting photons streaming freely in an expanding universe, the spectral distribution at a later stage would be

$$\rho_{\gamma\nu_0} d\nu_0 = d\nu_0 \, \nu_0^3 f(\nu_0 / T_0) \tag{90}$$

where

$$T_0 = TR/R_0 \tag{91}$$

is a scaled temperature inferred from an observed distribution and does not have any thermodynamic significance.

(d) Mean free estimates suggest that the observed radiation is not in thermal equilibrium with matter at present, whereas in the past an equilibrium state existed due both to ionization and higher matter density. It is therefore compelling to regard the observed radiation with a spectrum given by (90) as the remnant of the past.

We note that given the initial equilibrium distribution and the red-shift relation (86), the scaled radiation would keep the equilibrium form (90) if and only if the conservation law (88) holds. Since this is indeed the case in our theory, we conclude that the form of 3°K background radiation is unchanged.

(2) Friedman Equations. The Red Shift (Canuto et al., 1979b). Since in atomic units Einstein equations change because of the presence of terms, so will the corresponding Friedman equations. They in fact become

$$\left(\frac{\dot{R}}{R}+\frac{\dot{\beta}}{\beta}\right)^2+\frac{k}{R^2}=\frac{8\pi}{3}G\rho$$

$$\frac{\ddot{R}}{R}+\frac{\ddot{\beta}}{\beta}+\frac{\dot{\beta}\dot{R}}{\beta R}-\frac{\dot{\beta}^2}{\beta^2}=-\frac{4\pi}{3}G(3p+\rho) \tag{92}$$

Since the conservation laws are also altered by the presence of β, G factors,

$$\dot{\rho}+3\frac{\dot{R}}{R}(p+\rho)=-\rho\frac{(G\beta)^{\cdot}}{G\beta}-3p\frac{\dot{\beta}}{\beta} \tag{93}$$

we now have, for $p=c_s^2\rho$,

$$\rho\sim R^{-3(1+c_s^2)}G^{-1}\beta^{-1-3c_s^2} \tag{94}$$

which can then be used in (92).

At the same time we have

$$\overline{H}_0=H_0+h_0, \qquad h=\dot{\beta}/\beta$$

$$\rho_0/\rho_c=2\bar{q}_0\left(\overline{H}_0/H_0\right)^2 \tag{95}$$

where a bar indicates quantities in Einstein units.

For the gauge $G\beta^2=1$ and the $k=0$ case, the results simplify a great deal. We have for $G\sim t^{-1}$

$$R(t)\sim t^{1/2}, \qquad q_0=1$$

$$2H_0t_0=1, \qquad \overline{H}_0\bar{t}_0=2/3, \qquad \overline{H}_0=2H_0 \tag{96}$$

As expected, the age of the universe is larger in atomic units, $t_0=\frac{3}{2}\bar{t}_0$.

As for the red shift, we clearly have to introduce two symbols

$$1+z=R_0/R, \qquad 1+\varkappa=R_0/\overline{R} \tag{97}$$

where z is the physical atomic red shift, whereas $\varkappa$ is merely a mathematical symbol since it is related to the value of the scale factor in Einstein units. Between the two, there is evidently a simple relation

$$\beta(1+\varkappa)=1+z \tag{98}$$

which, for the specific case of $k=0$, reduces to

$$1+\varkappa=(1+z)^2 \tag{99}$$

which implies that a given, i.e., measured, z corresponds to a larger $\varkappa$.

(3) The m vs. z Relation (Canuto et al., 1979b). A series of rather delicate steps is required to derive the m vs. z relation, since several of the intervening relations are altered by the presence of β and G. We shall only quote the final result

$$l=\frac{L(t)}{4\pi R_0^2r_e^2}(1+z)^{-2}\beta^2G \tag{100}$$

where l and L are the apparent and absolute luminosity while the quantity r_e is given by the usual expression

$$\bar{q}_0^2\overline{H}_0\overline{R}_0(1+\varkappa)r_e=\bar{q}_0\varkappa+(\bar{q}_0-1)\left[(1+2\bar{q}_0\varkappa)^{1/2}-1\right] \tag{101}$$

where, however, we have to substitute $\varkappa$ vs. z before we can use it.

For the $\beta^2G=1$ case, we obtain for the m vs. z relation the simple expression for $k=0$,

$$m=m_0+5\log z+\tfrac{5}{2}e\log(1+z) \tag{102}$$

where we have taken the absolute luminosity to go like $L(t)\sim(1+z)^{-e}$.

In Figure 1 we plot m vs. z as from equation (102) against the most recent data. The solid line corresponds to standard cosmology with $\bar{q}_0 = 1$. The two dashed lines represent (100) for two values of $\bar{q}_0 = 0.1$ and 0.5. As one can easily see, the "degeneracy" among the values of $\bar{q}_0$ characterizing the standard framework is now lifted. Even a small difference in $\bar{q}_0$ yields quite different curves, thus allowing the possibility of determining $\bar{q}_0$ from this analysis, as originally thought by Sandage.

We may also note that a value of $\bar{q}_0 = 1$ would yield a theoretical curve bending too much toward the left. While an open universe is compatible with the data on QSOs, a closed universe does not seem to be favored by the present analysis.

(4) The Metric Angular Diameters (Canuto et al., 1979b). An equally important cosmological test concerns the largest angular diameter vs. z relation. It is easy to show that such an angle θ_m is given by

$$\theta_m \sim \beta(1+z)^2 \mathfrak{F}(z)^{-1} \tag{103}$$

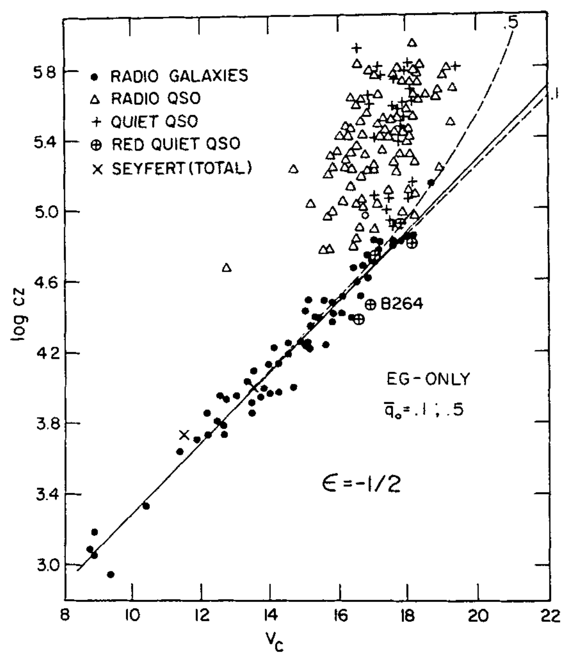

Fig. 1. The visual magnitude V_c vs. red shift, z. The solid curve corresponds to standard cosmology with $\bar{q}_0 = 1$. The two dashed curves give the predictions of the G-varying framework. The gauge chosen is $G\beta^2 = \text{const}$, with $G \sim 1/t$, i.e., $\beta(t) = (t/t_0)^{-\varepsilon}$, $\varepsilon = -1/2$.

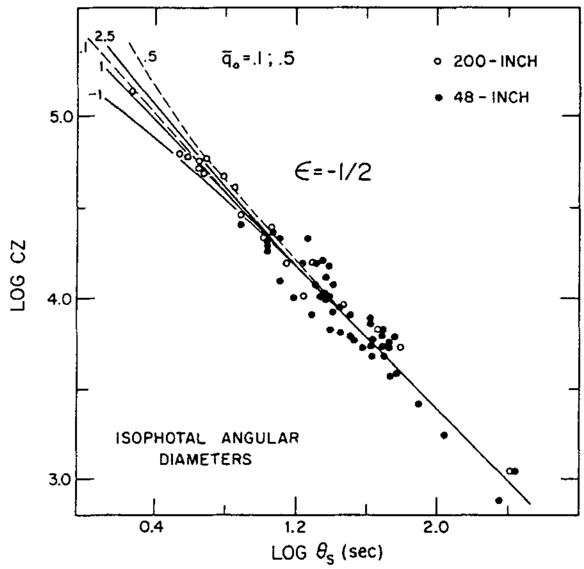

Fig. 2. The same as Figure 1 for isophotal angles θ_s. The solid curves correspond to standard cosmology with the value of $\bar{q}_0$ attached to each curve. The dashed lines correspond to a variable G. The value $\bar{q}_0 = -1$ corresponds to the steady-state cosmology.

For $\beta^2 G = \text{const}$, the results are shown in Figure 2, for the values $\bar{q}_0 = 0.1$ and 0.5. As in the case of Figure 1, the curves are more sensitive to $\bar{q}_0$ than in standard cosmology.

(5) The $\log N - \log S$ relation (Canuto and Owen, 1979). In the present cosmology, the derivation of the $N = N(S)$ relation is rather complex and we can only refer the reader to Canuto et al. (1979b) for details.

11. CONCLUSIONS

It is somewhat ironic that in spite of having shown that a violation of the SEP, or alternatively a time dependence of G, at the rate given by (15) does not contradict any known fact, we are still not entitled to conclude that we need such violation or that we have demonstrated its existence. It is in fact an intrinsic limitation of a theoretical approach to be at most able to prove *compatibility*. The proof of a variable G is in fact exclusively an observational question and we have indicated what the present available evidence is.

The theoretical and observational situations are, however, not entirely disconnected.

In fact, before embarking into a long and difficult search for a rather small effect, one needs to be reassured that such violation of the SEP is at least compatible with the already existing data. This was, however, not the general consensus before the present analysis was performed. In fact, the available indications then were that severe contradictions were likely to arise from a $G = G(t) \sim t^{-1}$.

It is difficult not to believe that such "popular wisdom," that persisted for many years, did not somewhat negatively influence those potentially interested in observing such effects. Our work has not shown that *G* varies. It has, however, shown, 40 years after Dirac first proposed it, that such variability is allowed and that in more than one instance it helps. It never worsens any fit to the data. We can therefore hope to have cleared the way for experimentalists to become interested in the subject so as to come soon to a final answer.

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